Mod 1- Part 1

Math Review

Making Measurements: Exact and Non-exact numbers

Exact numbers have no uncertainty in their value:

- Definitions: 12" = 1 foot
- Counted numbers

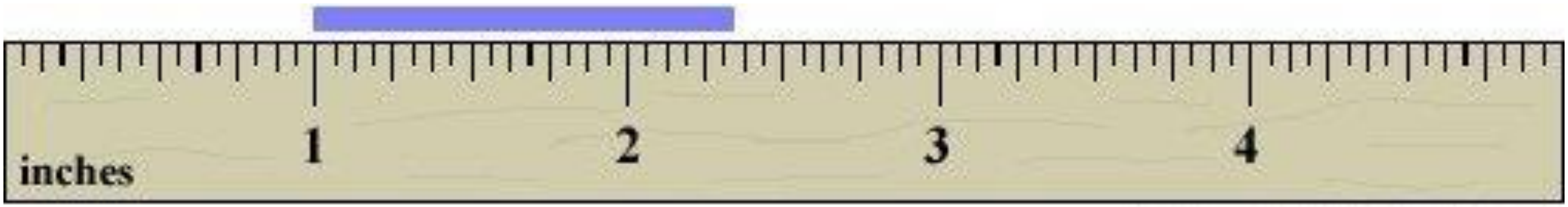
Non-exact numbers: Those obtained by measurement or observation. Always *some* uncertainty in them, because their value depends on the measuring instrument used.

Note: All measurements need a numerical value and a unit; *a quantity that describes the measurement being made.*

Making Measurements

Measuring length: English system

Line up object with 1st mark, not with end of ruler.



When measuring distance using a ruler marked off in inches, read to the 16th's of an inch place and then estimate the 32th's place

When measuring distance using a ruler marked off in centimeters, read to 0.1 cm place and then estimate the 0.01 cm place.

Significant Figure - A digit in a measurement that contributes to the measurement's precision. Significant figures include all measured digits plus one estimated digit.

There is always some error in the last sig fig of a measurement.

Rules of Significant Figures:

1. All non-zero figures (1, 2, 3, 4, 5, 6, 7, 8, 9) are significant.
2. A zero is significant if it is between two significant figures.
3. A zero is also significant if it is both at the end of the number and to the right of the decimal point.

The Atlantic-Pacific Rule for determining Sig Figs

<u>Pacific</u>

If a decimal is present, start counting at the left side of the number, and, beginning with the first non-zero number, count all the numbers.

<u>Atlantic</u>

If a decimal is absent, start counting at the right side of the number and, beginning with the first non-zero number, count all the numbers.

How many significant figures are in 102.0 inches?

How many significant figures are in 0.0405 cm?

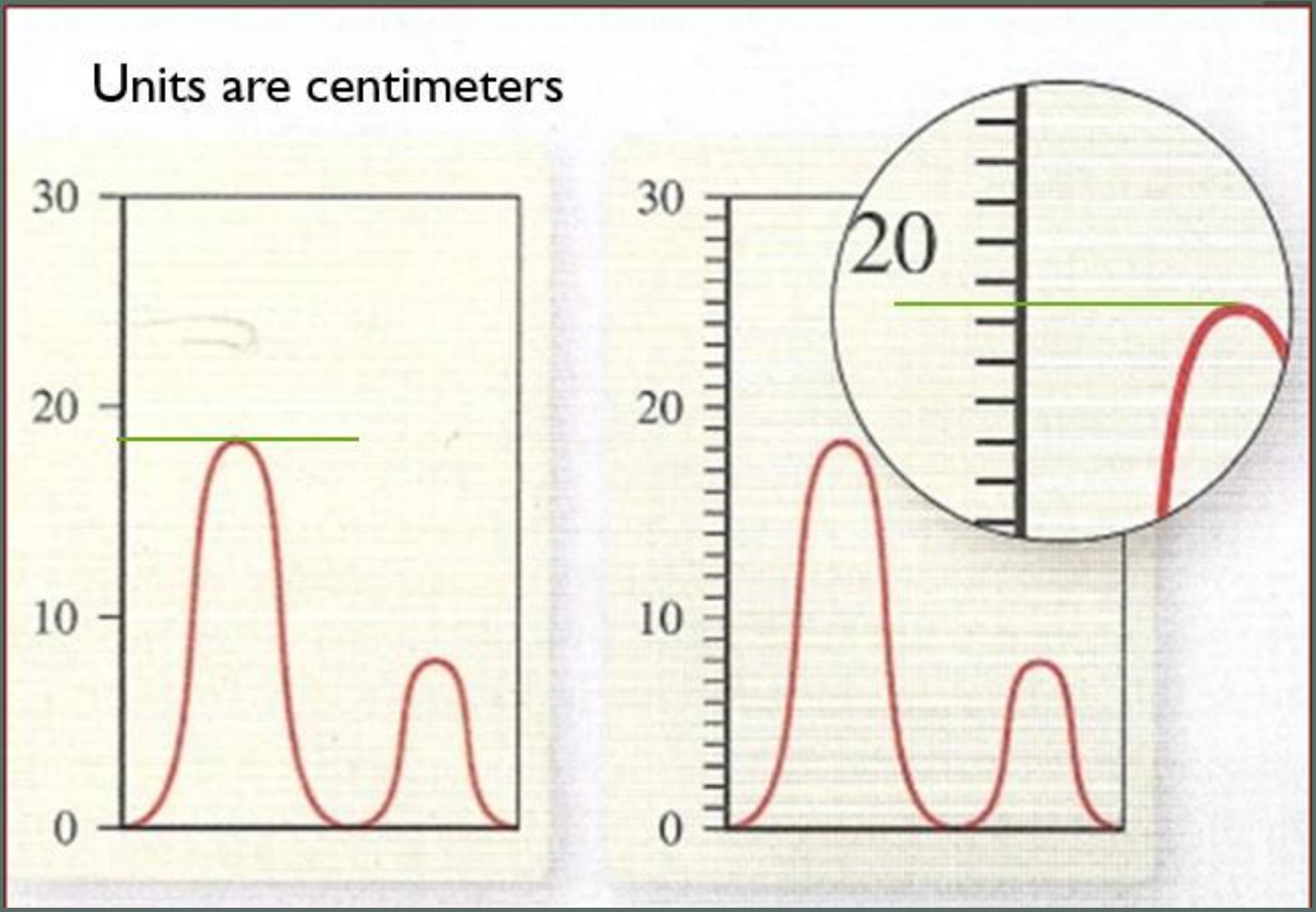
How many significant figures are in 12,000 inches?

How many significant figures are in each of the numbers below?

0.01020 in   12,007 cm   609,000 cm   89.10400 in

Precision is indicated by the number of significant figures.

Significant figures: All digits of which we are certain, plus one estimated digit



1st graph

Estimate: 18 cm

Gradations every 10 cm. Estimate units' place.

2nd graph

Estimate: 18.4 cm

Gradations every 1 cm.

Estimate tenths' place

Sheep dog to owner: “I rounded up all 400 of the sheep.”
Owner, counting them: “But there are only 387.”

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Owner, counting them: "But there are only 387."
Sheep dog: "Yes, I rounded them up."

Remember how to round numbers.

ROUNDING NUMBERS

Rounding numbers: If a number contains more digits than are allowed by the rules of significant figures, drop the digits after the last sig fig, using this procedure:

If the first digit being dropped is five or higher, increase the last significant figure by 1 unit. If the first digit being dropped is 4 or lower, leave the last significant figure unchanged. i.e.:

Rounding to three sig figs:

6.077 becomes 6.08

but 6.073 becomes 6.07

Say you have two pieces of wood.

You measure one as 12.12 cm long, and another person measures the other, with a less-precise ruler, as 9.5 cm long.

You glue those pieces of wood together and want to get the new length without measuring it.

So, you add the two lengths together, and get:

12.12 cm + 9.5 cm = 21.62 cm

$$\begin{array}{r} 12.12\text{ cm} \\ +\ \ 9.5\phantom{2\text{ cm}} \\ \hline 21.62\text{ cm} \end{array}$$

12.12 cm + 9.5 cm = 21.62 cm

But your first measurement (12.12 cm) is more precise than the second one.

When adding and subtracting measurements, you must report your answer to the same precision as the *least precise number* in the problem.

Must round answer to tens’ place: 21.6 cm

$$\begin{array}{r} 12.12\text{ cm} \\ +\ \underline{9.5\text{ cm}} \\ 21.62\text{ cm} \rightarrow 21.6\text{ cm} \end{array}$$

What do you get if you subtract 15.423 cm from 102 cm?

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$$
\begin{array}{r}
102 \qquad \text{cm} \\
\underline{-\ \ 15.423\ \text{cm}} \\
86.577\ \text{cm}
\end{array}
$$

102 cm has its last significant figure in the ones' place. It is less precise than 15.423, which has its last significant figure in the thousandths' place.

So, 102 cm limits the precision of the answer to the ones' place, so the correct answer is 87 cm. We also had to round up, because of the "5" in the tenths' place.

We have different rules for multiplying and dividing:

When multiplying and dividing measurements, your answer must have the *same number of significant figures* as the measurement which has the fewest significant figures. So, here we are not concerned about place value.

$$\frac{15.423 \text{ cm}}{102 \text{ cm}} =$$

$$\begin{array}{r} 4.209 \text{ in} \\ \underline{\times\ 7.0 \text{ in}} \end{array}$$

We have **different rules** for multiplying and dividing:

When multiplying and dividing measurements, your answer must have the *same number of significant figures* as the measurement which has the fewest significant figures. So, here we are not concerned about place value.

$$\frac{15.423 \text{ cm}}{102 \text{ cm}} = 0.151205882353 \quad = 0.151$$

$$\begin{array}{r} 4.209 \text{ in} \\ \underline{\times\ 7.0 \text{ in}} \\ 29.463 \text{ in}^2 \end{array} \quad = 29 \text{ in}^2$$

USE SCIENTIFIC NOTATION TO **MAKE** FIGURES SIGNIFICANT

The number 7,645 is identical to 7.645×10^3, so it can be reported using either decimal or scientific notation.

The only time scientific notation *must* be used is when the significant figures can only be properly expressed with scientific notation;

$125 \times 6.4 = 800$

Answer needs two sig figs, so express as: 8.0×10^2

3.1 in + 8.991 in

45.5 cm x 9 cm

3.1 in + 8.991 in

12.091 = 12.1 in

45.5 cm x 9 cm

409.5 cm^2 = 400 cm^2

Two students measure the mass of an object. The teacher knows that the mass is 4.5 g. The first student uses one scale and reports a mass of 4.4 g. The second student uses a different scale and reports a mass of 4.210 g. Which student was more precise? Which student was more accurate?

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The second student was more precise, and the first student was more accurate.

Units of Measurement : The Metric (S.I.) System

"Metrics is for all people, for all time." King Louis XVI

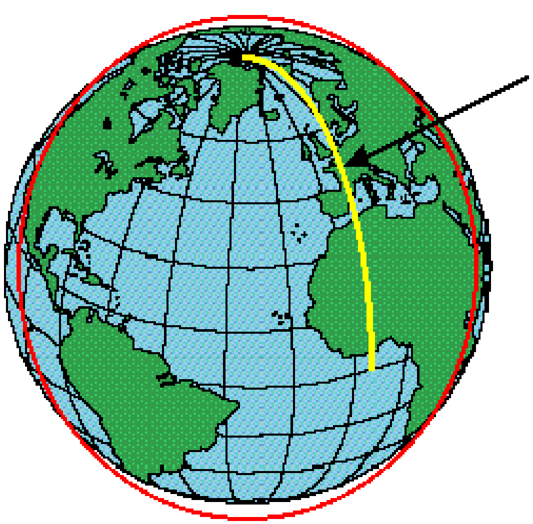
Based on powers of ten. Prefixes multiply or divide units.

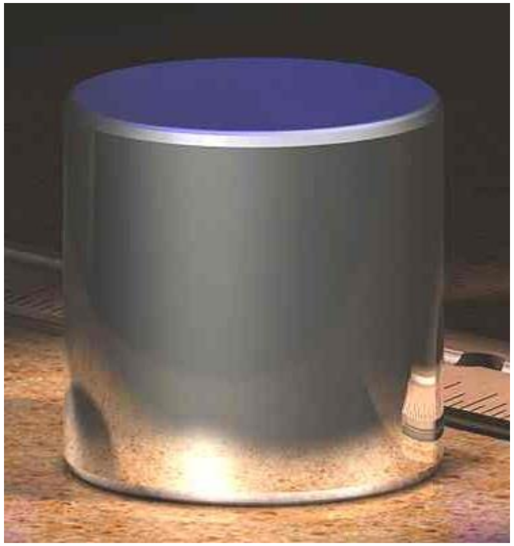
Meter: from Greek word *Metron:* "to measure."

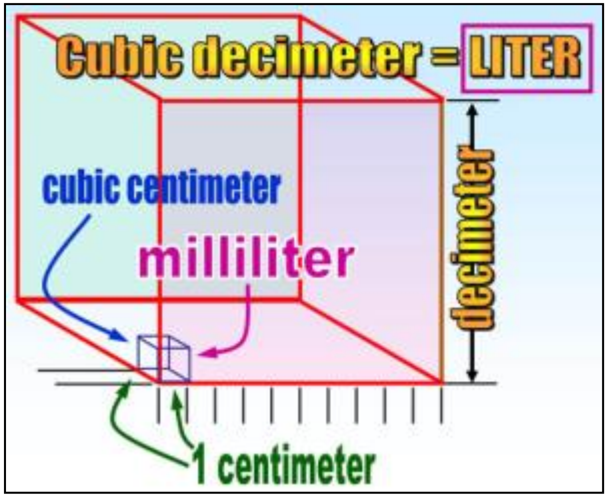
One ten-millionth of the distance of a line extending from the Equator to the North Pole passing through Paris, France.

Kilogram: the mass of a liter of water at 4 °C and 760 mm of Hg pressure.

Liter: The volume of 1 kilogram of water; 1/1,000 of a cubic meter; equals a cubic decimeter; 1,000 mL.







Metric Prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

Prefix	Meaning
micro (μ)	0.000001
milli (m)	**0.001**
centi (c)	**0.01**
deci (d)	0.1
deca (D)	10
hecto (H)	100
kilo (k)	**1,000**
Mega (M)	1,000,000

Metric prefixes mean the same thing regardless of the quantity being measured; i.e., mass, volume, distance, time.

Kilometer = 10^3 meters

Length of about 5 city blocks

Meter = Distance from door knob to floor

Decimeter = 10^{-1} m

Size of a large orange

Centimeter = 10^{-2} m

Width of a shirt button

Millimeter = 10^{-3} m

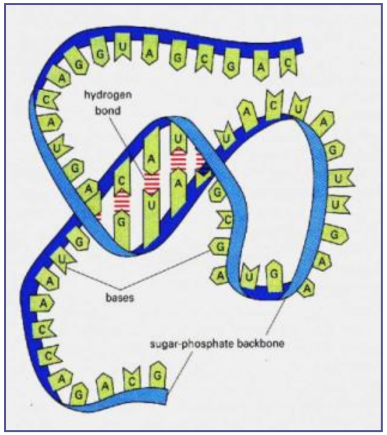
Thickness of a dime

Micrometer = 10^{-6} m

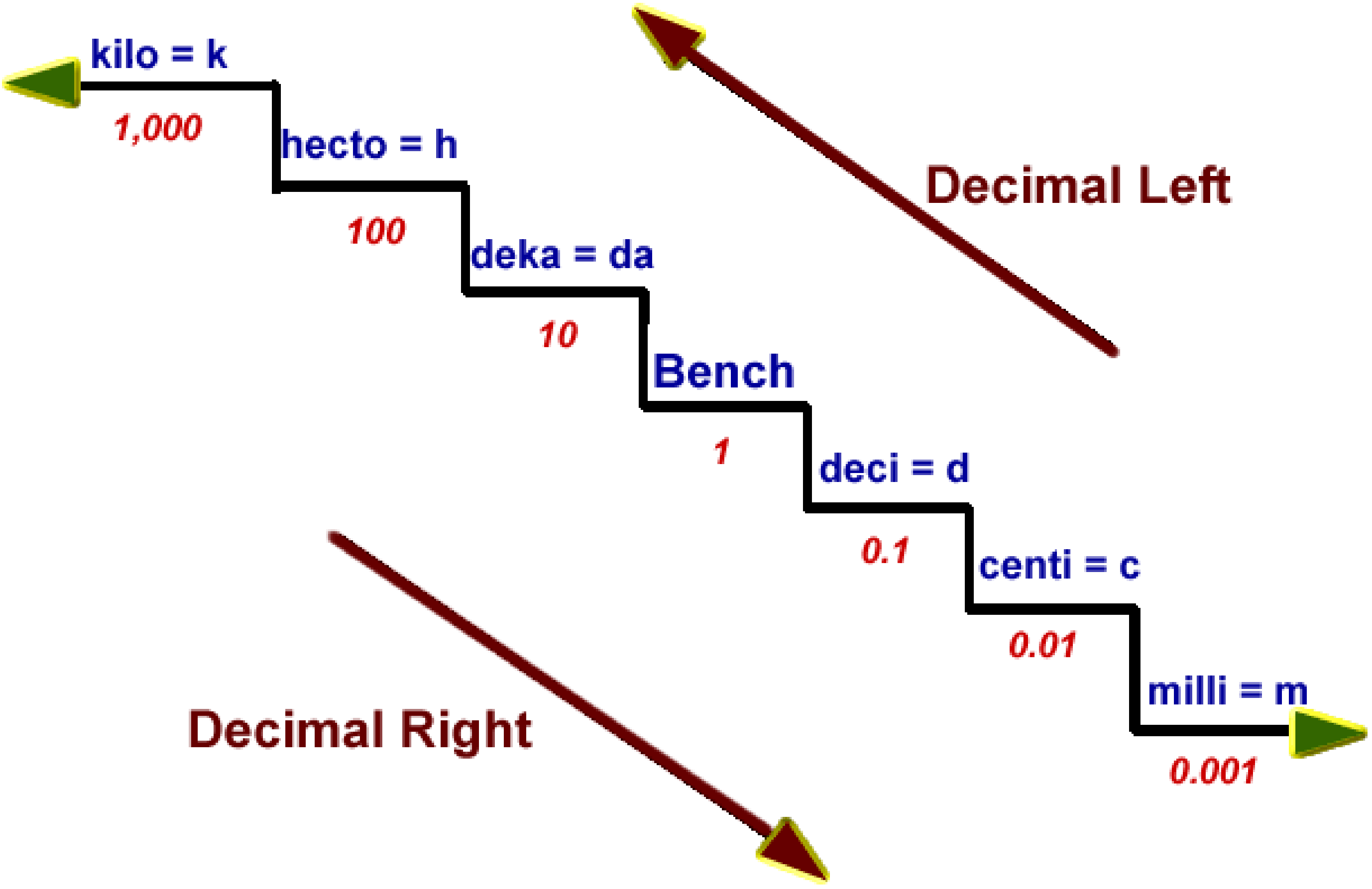
Diameter of bacterial cell

Nanometer = 10^{-9} m

Thickness of an RNA molecule



The Metric Staircase



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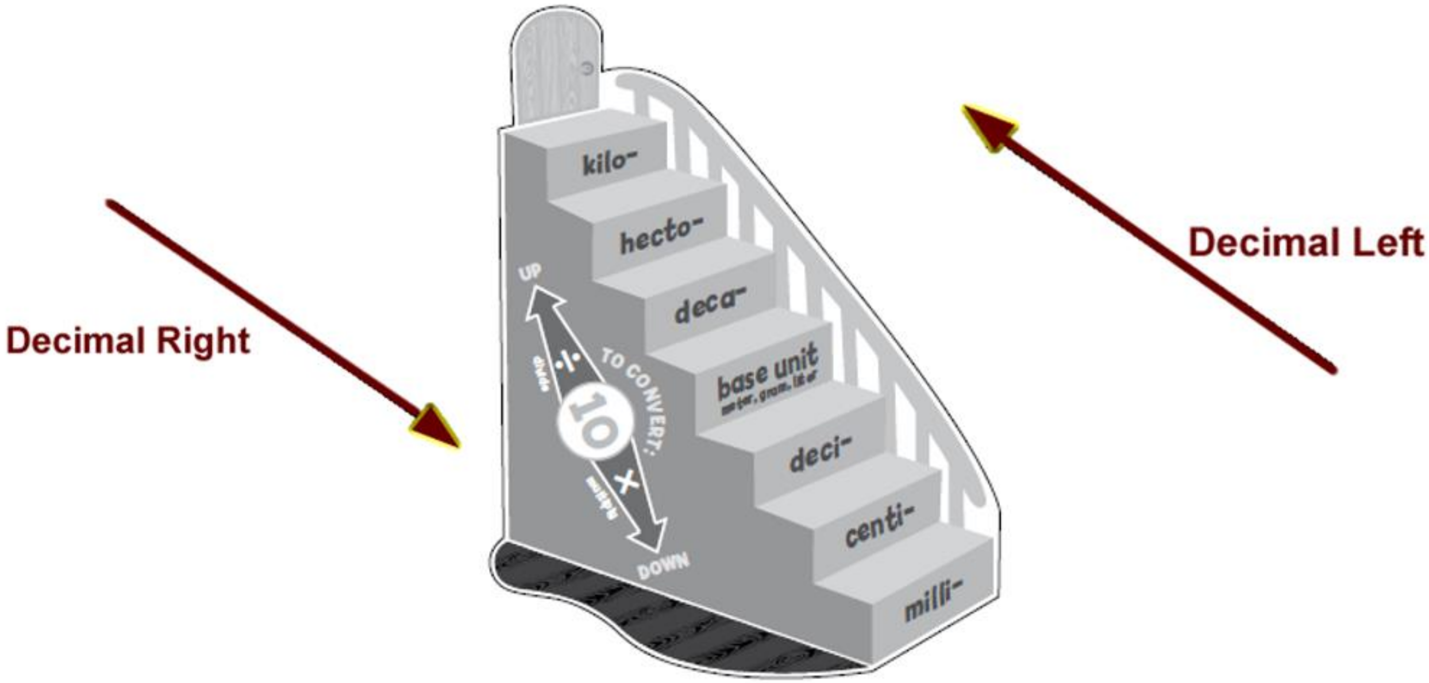

Don't be confused by the fact that a millimeter is $\frac{1}{10}$ of a centimeter, yet the arrow on the staircase says to multiply as you go down the staircase. This is because the arrows are for converting metric units to make them equivalent. For example, if you have 1 cm and you are trying to find its equivalent in millimeters, you must multiply by 10 (move the decimal to the right) to find the number of (smaller) millimeters which equals the single (larger) centimeter.

Converting between units- Dimensional Analysis (factor-label method)

First, write the measurement you know, in fraction form, by putting it over one.
Next, find the relationship between the unit you have and the unit you want to convert to. Use that relationship to create a fraction that, when multiplied by the first fraction, cancels out the unit you have and replaces it with the unit you want to have; e.g.

Convert 15.1 cm to meters:

$$\frac{15.1\ cm}{1}\ x\ \boxed{\frac{1\ meter}{100\ cm}} = 0.151\text{ m}$$

If a rock weighs 14,351 grams, how many kg does it weigh?

If a rock weighs 14,351 grams, how many kg does it weigh?

$$\frac{14{,}351\ \text{g}}{1} \times \frac{1\ \text{kg}}{1000\ \text{g}} =$$

$$14.351\ \text{kg}$$

Converting between unit systems-

Measurement	English/Metric Relationship
Distance	1 inch = 2.54 cm
Mass	1 slug = 14.59 kg
Volume	1 gallon = 3.78 L

If a tabletop is 37.8 inches, how many cm is that?

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If a tabletop is 37.8 inches, how many cm is that?

$$\frac{37.8\text{ in}}{1} \times \frac{2.54\text{ cm}}{1\text{ in}} =$$

96.012 cm rounded to 96.0 cm

More complex conversions

How many kl equal 4,523 cl?

If the mass of an object is 0.030 kg, what is that mass in mg?

More complex conversions

How many kl equal 4,523 cl?

$$\frac{4{,}523\ \text{cl}}{1} \times \frac{1\ \text{l}}{100\ \text{cl}} \times \frac{1\ \text{kl}}{1000\ \text{l}} =$$

0.04523 kl

If the mass of an object is 0.030 kg, what is that mass in mg?

$$\frac{0.030\ \text{kg}}{1} \times \frac{1{,}000\ \text{g}}{1\ \text{kg}} \times \frac{1{,}000\ \text{mg}}{1\ \text{g}} =$$

30,000 mg, reported out as 3.0×10^4 mg

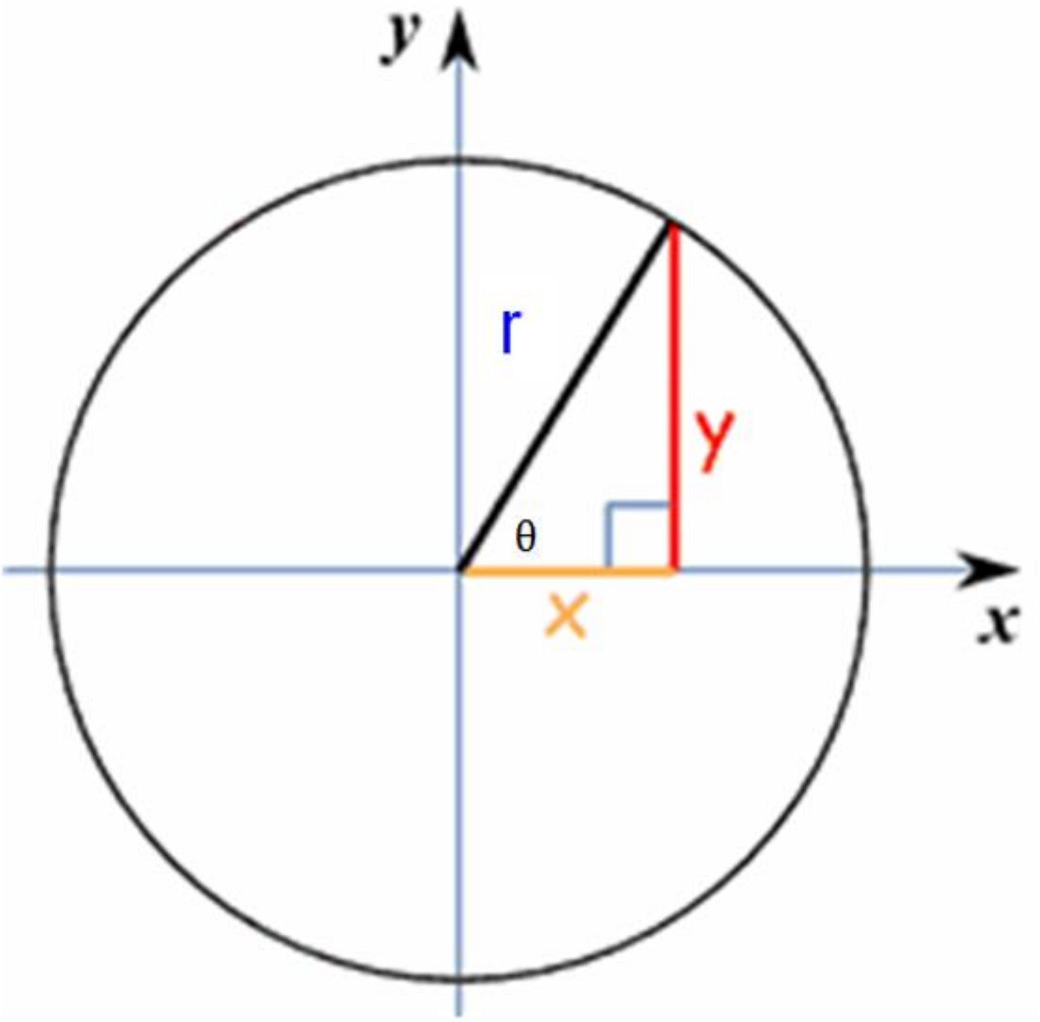
You must know the definitions of, and be able to calculate, these trigonometry functions:

$y / r = \sin\theta$

$x / r = \cos\theta$

$y / x = \tan\theta$

You must know how to find their inverses (the angles that have those functions) on a calculator.



Scientific Notation

Measurement: a quantity with both a number and a unit.

In chemistry, we encounter very LARGE and very small numbers.

A single gram of hydrogen, contains approximately 602,000,000,000,000,000,000,000 hydrogen atoms. That is ~*602 sextillion atoms!*

You can work more easily with very large or very small numbers by writing them in scientific notation.

In scientific notation, a given number is written as the product of two numbers: a coefficient and 10 raised to a power:

i.e., 4.78×10^5

Scientific Notation

For example, the number 602,000,000,000,000,000,000,000 can be written in scientific notation as 6.02×10^{23}.

The coefficient is 6.02. The power of 10, or exponent, is 23.

In scientific notation, the coefficient is always a number *greater than or equal to one and less than ten.* The exponent is an integer.

A positive exponent indicates how many times the coefficient must be multiplied by 10.
A negative exponent indicates how many times the coefficient must be divided by 10.

Scientific Notation

Numbers > 1 have a positive exponent, which equals the number of places that the original decimal point has been moved to the left.

$6,300,000. = 6.3 \times 10^{6}$

$94,700. = 9.47 \times 10^{4}$

Numbers between zero and one have a negative exponent. The value of the exponent equals the number of places the decimal has been moved to the right.

$0.000\ 008 = 8 \times 10^{-6}$

$0.00736 = 7.36 \times 10^{-3}$

Multiplying and Dividing in Scientific Notation

To multiply numbers written in scientific notation, multiply the coefficients and add the exponents.

$(3 \text{ x } 10^4) \text{ x } (2 \text{ x } 10^2) =$

$(2.1 \text{ x } 10^3) \text{ x } (4.0 \text{ x } 10^{-7}) =$

Multiplying and Dividing in Scientific Notation

To multiply numbers written in scientific notation, multiply the coefficients and add the exponents.

$(3 \times 10^{4}) \times (2 \times 10^{2}) = (3 \times 2) \times 10^{4+2} = 6 \times 10^{6}$

$(2.1 \times 10^{3}) \times (4.0 \times 10^{-7}) = (2.1 \times 4.0) \times 10^{3+(-7)}$

$= 8.4 \times 10^{-4}$

Multiplying and Dividing in Scientific Notation

To divide numbers written in scientific notation, divide the coefficients and subtract the exponent in the denominator from the exponent in the numerator.

$$\frac{3.0 \times 10^5}{6.0 \times 10^2} =$$

Multiplying and Dividing in Scientific Notation

To divide numbers written in scientific notation, divide the coefficients and subtract the exponent in the denominator from the exponent in the numerator.

$$\frac{3.0 \times 10^5}{6.0 \times 10^2} = \left(\frac{3.0}{6.0}\right) \times 10^{5-2} = 0.5 \times 10^3 = 5.0 \times 10^2$$

Adding and Subtracting in Scientific Notation

To add or subtract numbers expressed in scientific notation *the exponents must be the same*.

The decimal points *must be aligned* before you add or subtract the numbers.

$$(5.4 \times 10^3) + (8.0 \times 10^2)$$

Adding and Subtracting in Scientific Notation

To add or subtract numbers expressed in scientific notation *the exponents must be the same*.

The decimal points *must be aligned* before you add or subtract the numbers.
When adding 5.4×10^3 and 8.0×10^2, first rewrite the second number so that the exponent is a 3. Always convert from the smaller to the larger exponent. Then add the numbers.

$$\begin{aligned}(5.4 \times 10^3) + (8.0 \times 10^2) &= (5.4 \times 10^3) + (0.80 \times 10^3)\\ &= (5.4 + 0.80) \times 10^3\\ &= 6.2 \times 10^3\end{aligned}$$

Solve each problem and express the answer in scientific notation.

a. $(8.0 \times 10^{-2}) \times (7.0 \times 10^{-5})$

b. $(7.1 \times 10^{-2}) + (5 \times 10^{-3})$

Multiply the coefficients and add the exponents.

a. $(8.0 \times 10^{-2}) \times (7.0 \times 10^{-5}) = (8.0 \times 7.0) \times 10^{-2 + (-5)}$

$= 56 \times 10^{-7}$

$= 5.6 \times 10^{-6}$

b. $(7.1 \times 10^{-2}) + (5 \times 10^{-3}) = (7.1 \times 10^{-2}) + (0.5 \times 10^{-2})$

$= (7.1 + 0.5) \times 10^{-2}$

$= 7.6 \times 10^{-2}$